Department of
Engineering Physics
UNIVERSITY OF WISCONSIN–MADISON

Waves in Plasmas

by Steffi Diem with cited references

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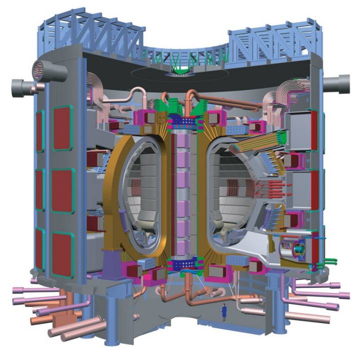
Introduction to Fusion Energy
and Plasma Physics Course

SULI, June 14th, 2022

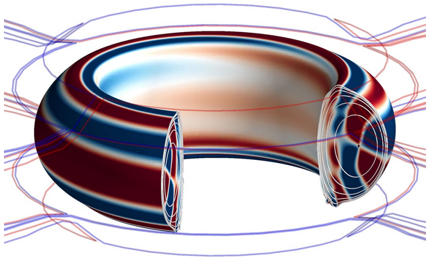
Webpage: https://directory.engr.wisc.edu/ep/Faculty/Diem_Stephanie/

Recent advances have made great progress towards practical, economical fusion energy

ITER: Access to burning plasma

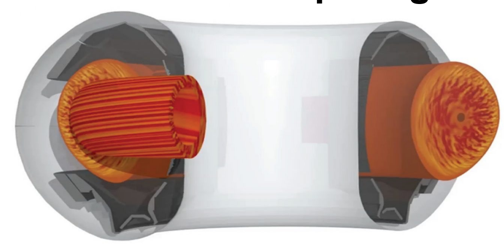


Advanced plasma control



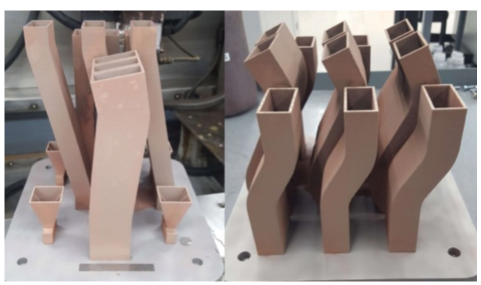
J-K Park, PPPL

Exascale computing



A. Bhattacharjee

Additive/Advanced manufacturing



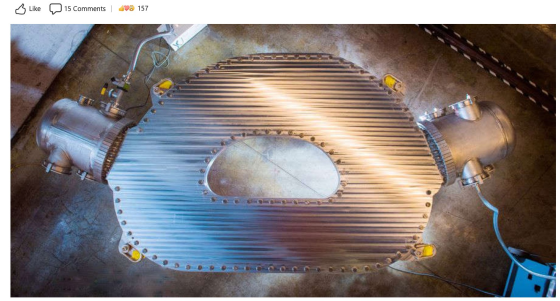
A. Seltzman, APS-DPP (2020)

Hight temperature superconductors

POPULAR MECHANICS

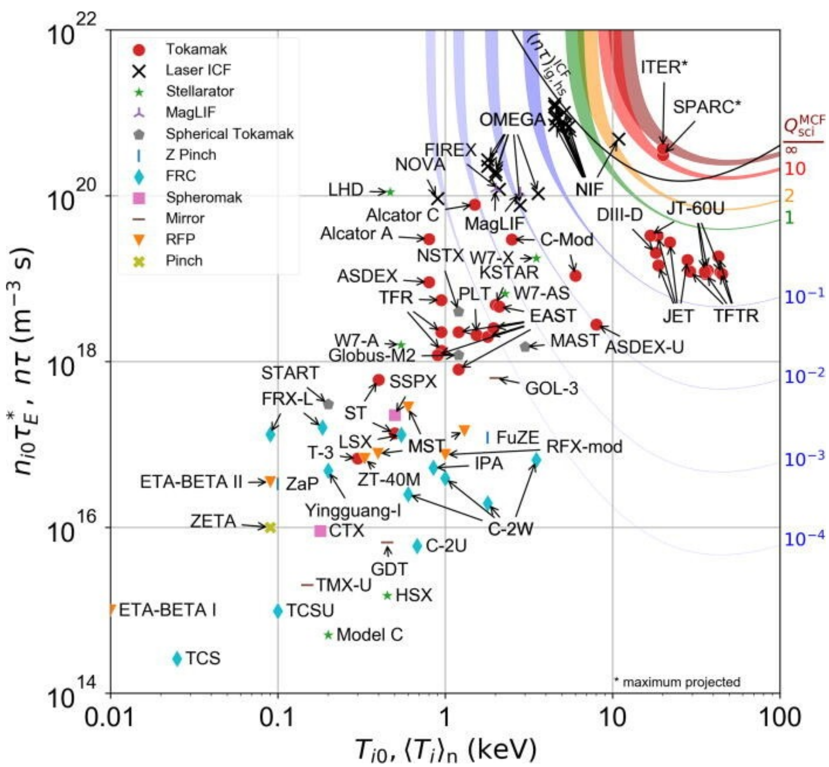
Scientists Just Generated the World's Strongest Magnetic Field

Caroline Delbert · 10/27/2021



Commonwealth Fusion Systems

We've been making great progress in confining plasmas



S.E. Wurzel and Scott C. Hsu, Physics of Plasmas 29, 062103 (2022)

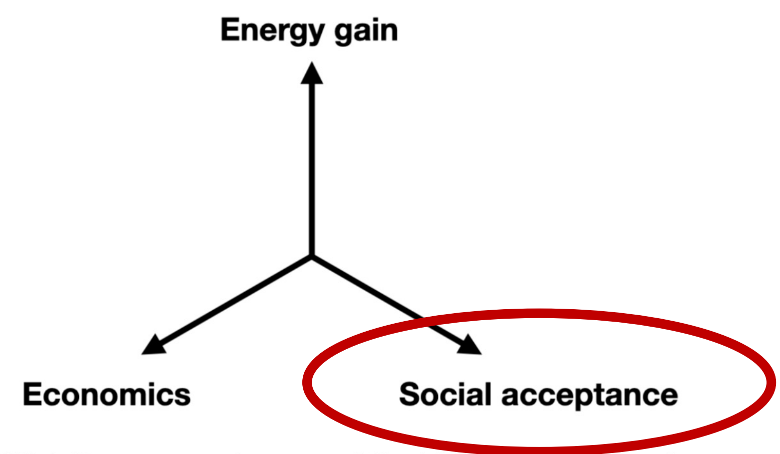


FIG. 1. Progress towards commercially viable fusion energy requires progress along three equally important axes. This paper focuses only on the axis of energy gain.

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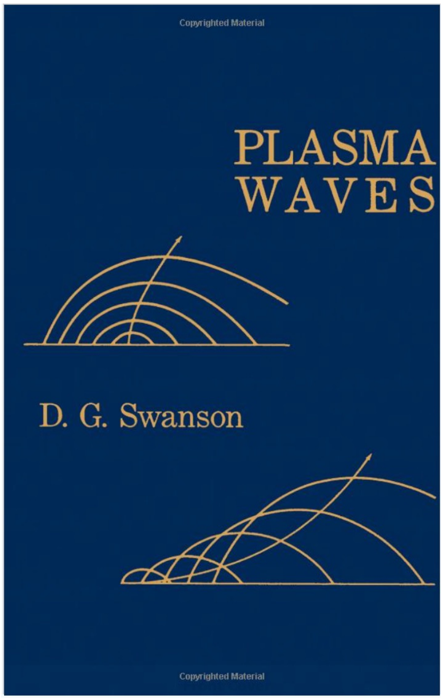
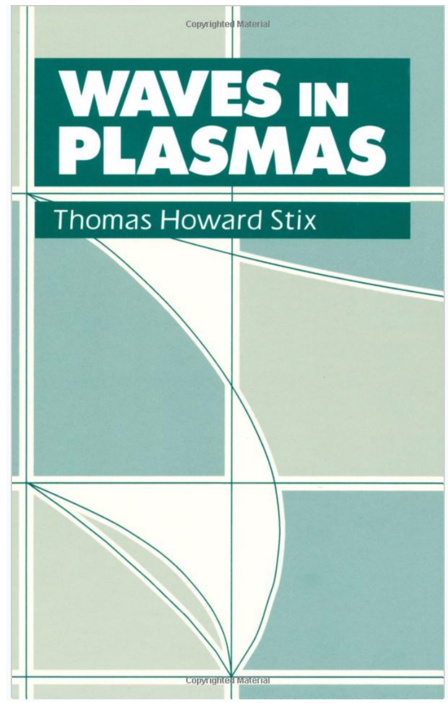
Waves References



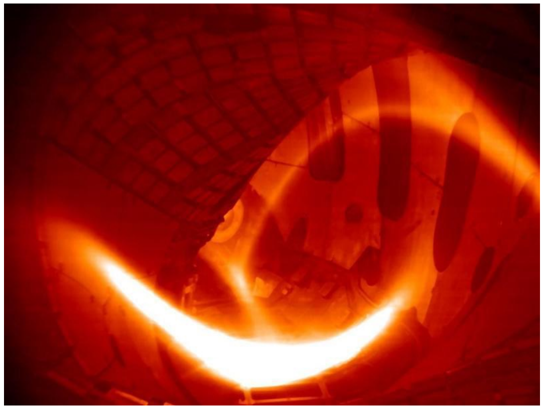


My SULI talk from a previous year, along with many other cited references in this talk..

Plasmas support wide variety of wave phenomena

- Waves found naturally in plasmas
  – Instabilities, fluctuations, wave-induced transport
- Waves can deliver energy-momentum in plasma
  – Heating, current drive, particle acceleration
  – Mode stabilization, plasma confinement, α-channeling
- Waves can be used in plasma diagnostics
  – Interferometry, reflectometry, Faraday rotation, Thomson scattering

Photo of aurora: Senior Airman Joshua Strang



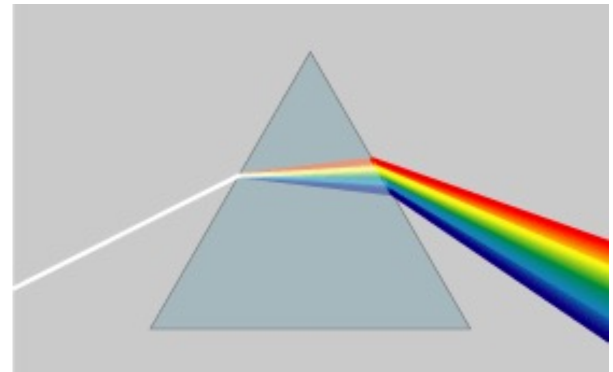
First W7-X plasma, IPP, Greifswald

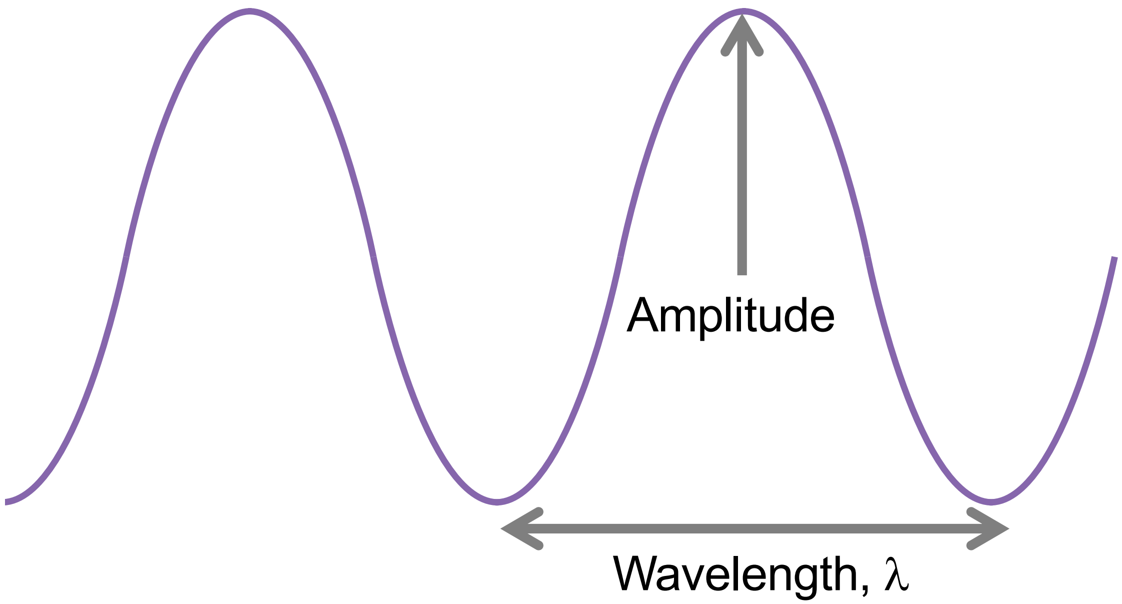
Plasmas support wide variety of wave phenomena

- **How do we describe waves in plasmas?**
- What can the dispersion relation tell us?
- Examples of waves and what we can do with them

How are waves in plasmas described?

- Wave characteristics can change based on surroundings
- Dispersion relation describes relationship between wavelength and frequency of wave, **ω(k)**



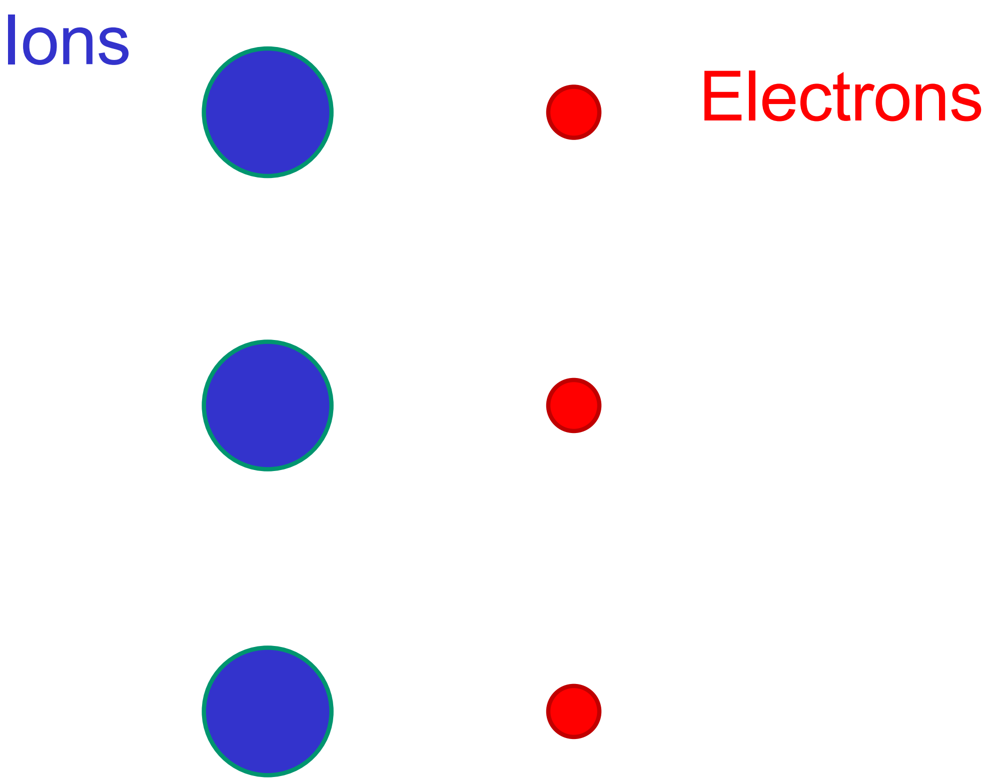


Wavenumber: $k=2\pi/\lambda$

Angular frequency: $\omega=2\pi f$

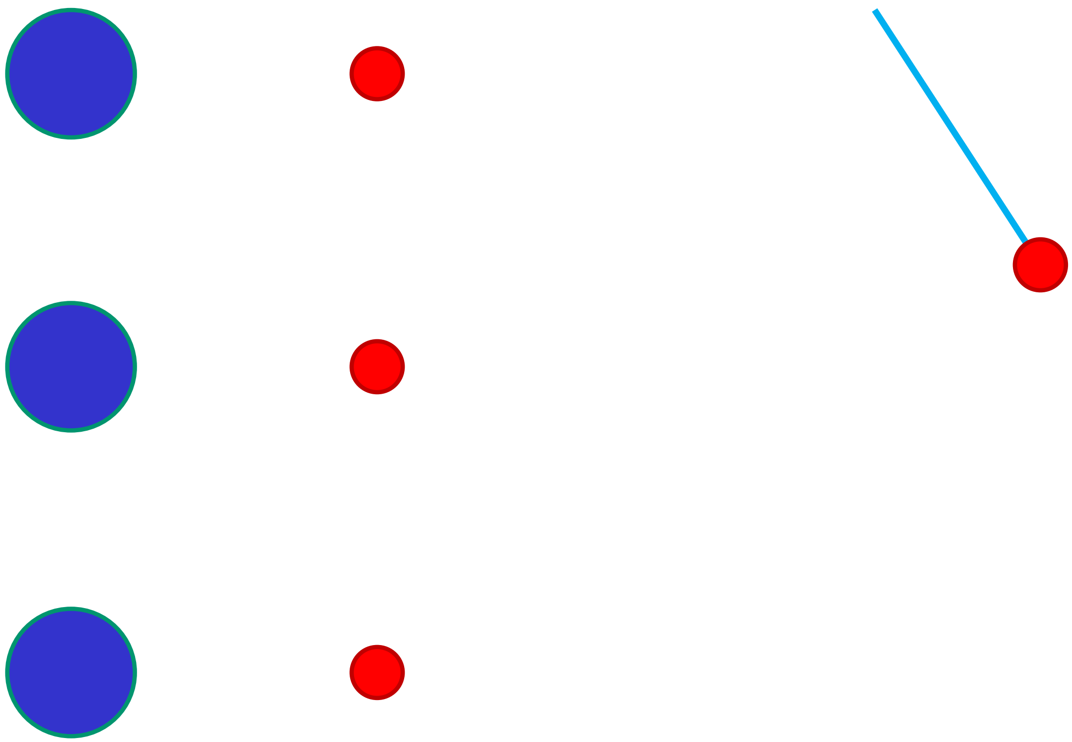
Phase velocity: $v_p=\omega/k$

Plasma wave in a cold plasma

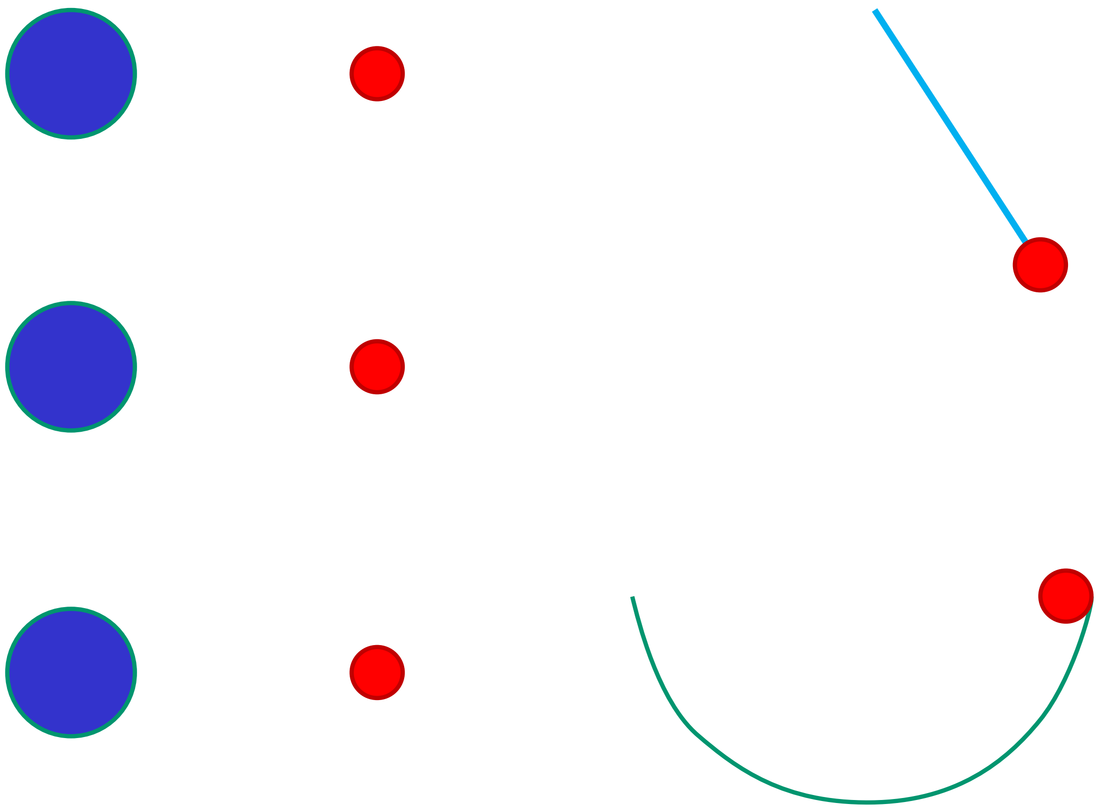


Example from [Prof. Mordijck's 2017 SULI talk]

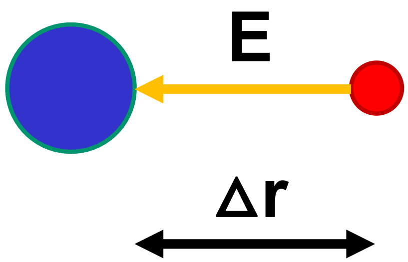
Plasma wave: Similar to pendulum motion



Plasma wave: Similar to ball stuck in a valley



Plasma wave: Plasma frequency



$$m_e \frac{dv}{dt} = -eE$$

$$E \sim \frac{q}{\Delta r^2} = \frac{e n_e}{\Delta r^2}$$

$$\frac{d^2 r}{dt^2} = -\frac{e^2 n_e}{m_e} \frac{1}{\Delta r^2}$$

Compare to a simple harmonic oscillator:

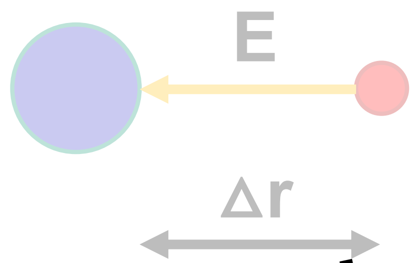
$$\frac{d^2 r}{dt^2} = -\omega r$$

Assume that Δr ~1, then:

$$\frac{1}{\Delta r^2} \sim \Delta r$$

$$\boldsymbol{\omega_p \sim \frac{n_e e^2}{m_e}}$$

Plasma wave: Plasma frequency



Compare to a simple harmonic oscillator:

$$\frac{d^2r}{dt^2} = -\omega$$

Assume that $\Delta r \sim 1$, then:

$$m_e \frac{dv}{dt} = -eE$$

$$\frac{1}{\Delta r^2} \sim \Delta r$$

$$E \sim \frac{q}{\Delta r^2} = \frac{e n_e}{\Delta r^2}$$

$$\omega_p \sim \frac{n_e e^2}{m_e}$$

$$\frac{d^2r}{dt^2} = -\frac{e^2 n_e}{m_e}\frac{1}{\Delta r^2}$$

Wide variety of waves can exist in plasmas, requires a more complex description

For a more complete cold plasma wave derivation, see my [2019 SULI presentation]()

How are waves in plasmas described?

- Plasmas respond to magnetic fields, plasmas conduct electricity
- Process to derive plasma dispersion relations
    1. Determine assumptions

Is there an external magnetic field and is it uniform?

Are the ions or electrons cold?

What frequency range are you looking at?

Is one species hot?

How are waves in plasmas described?

- Plasmas respond to magnetic fields, plasmas conduct electricity
- Process to derive plasma dispersion relations
  1. Determine assumptions
  2. Fourier analyze Maxwell's equations to obtain wave equation

Apply Fourier analysis in space and time: $\vec{E},\vec{B} \approx \exp\left(i\vec{k} \bullet \vec{r} - i\omega t\right)$

How are waves in plasmas described?

- Plasmas respond to magnetic fields, plasmas conduct electricity
- Process to derive plasma dispersion relations
  1. Determine assumptions
  2. Fourier analyze Maxwell's equations to obtain wave equation
  3. Obtain dielectric tensor, relates plasma current to electric field
  4. Combine the above to yield the dispersion relation, ω(k)

Dielectric tensor – ***contains all of the plasma physics***

$$\vec{n} \times \vec{n} \times \vec{E} + \overline{\overline{K}} \bullet E = 0$$

For **K** = 1, get vacuum waves

Index of refraction: $\vec{n} = \dfrac{c\vec{k}}{\omega}$

How are waves in plasmas described?

- Plasmas respond to magnetic fields, plasmas conduct electricity
- Process to derive plasma dispersion relations
  1. Determine assumptions
  2. Fourier analyze Maxwell's equations to obtain wave equation
  3. Obtain dielectric tensor, relates plasma current to electric field
  4. Combine the above to yield the dispersion relation, ω(k)
- This process only results in waves in plasmas
  - No resulting instabilities because there are no sources
  - Provides basic framework for how more complex dispersion relations are derived

$$\vec{n} \times \vec{n} \times \vec{E} + \overline{\overline{K}} \bullet E = 0$$

For **K** = 1, get vacuum waves

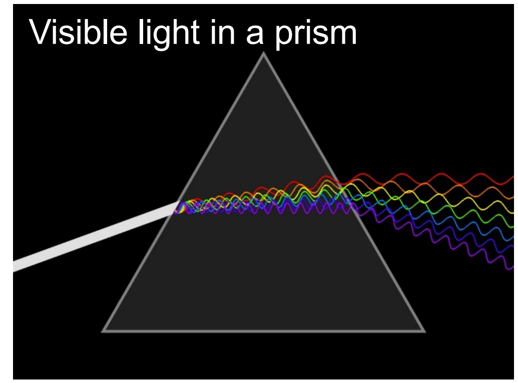
Index of refraction: $\vec{n} = \frac{c\vec{k}}{\omega}$

Plasmas support wide variety of wave phenomena

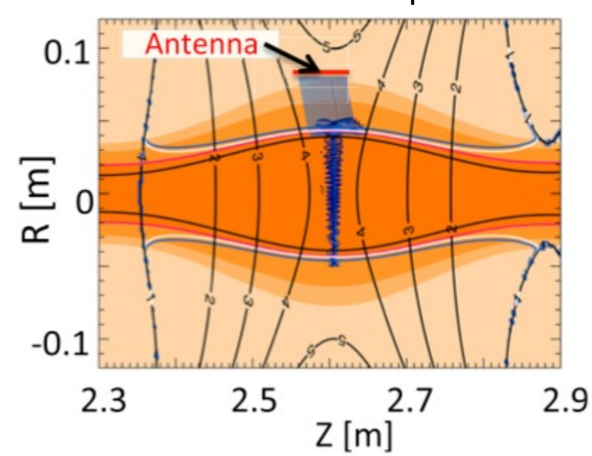
- How do we describe waves in plasmas?
- **What can the dispersion relation tell us?**
- Examples of waves and what we can do with them

Dispersion relation contains lots of information

- Wave characteristics can change based on surroundings
- Dispersion relation describes relationship between wavelength and frequency of wave, $\omega(k)$
- Resonances can occur when $n \rightarrow \infty$
  - At this point, the wavelength goes to zero
  - The waves resonant with particle motion, phase velocity goes to zero and energy can be absorbed
- Cutoffs occur when $n \rightarrow 0$
  - At this point, the wavelength goes to infinity
  - The waves will not propagate and typically significant reflection occurs



Microwaves in a plasma



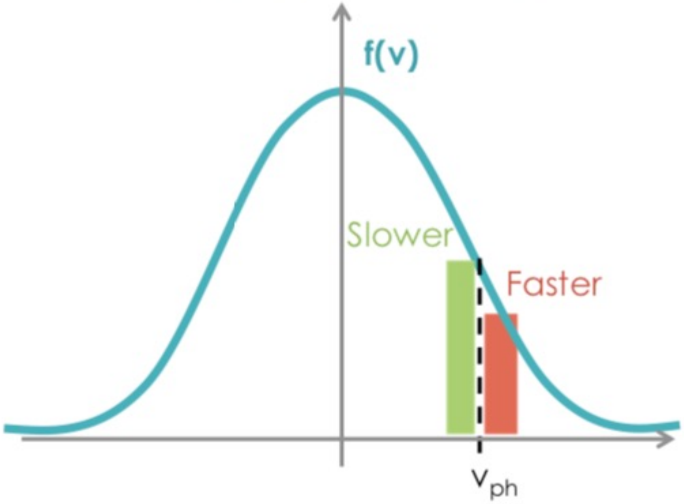
Diem et al, Phys. Plasmas (2018)

Plasmas support wide variety of wave phenomena

- How do we describe waves in plasmas?
- What can the dispersion relation tell us?
- **Examples of waves and what we can do with them**

Interactions of waves and particles

- Collisionless damping allows energy exchange between plasma and electromagnetic waves
- Particles with speed comparable to v_{ph} speed can resonate
- Particles with speed slightly slower than v_{ph} will be accelerated, take energy from wave
- Particles with speed slightly faster than v_{ph} will decelerate, give energy to wave



Waves in cold plasma dispersion relation

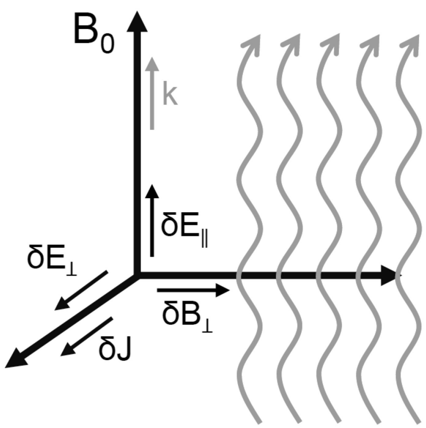
- Several waves can be solved for with the cold plasma dispersion relation
  - Only a few examples will be shown in this talk

- Propagation parallel to B_0
  - Plasma oscillations (first example)
  - **Alfven waves**
  - Whistler waves

- Propagation perpendicular to B_0
  - Ordinary and extraordinary waves

Low frequency MHD waves – Alfvén waves

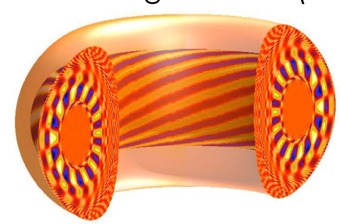
- Very low frequency waves ($\omega << \Omega_{CI}$)
- Magnetohydrodynamic (MHD) wave where ions oscillate in response to a restoring force provided by an effective tension on the magnetic field lines
  - Linearize MHD equations to obtain shear Alfvén
  - EM waves that propagate along magnetic field lines



$$\omega = k_{\parallel} v_A$$

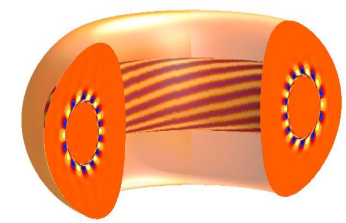
$$v_A = \frac{B}{\sqrt{\mu_0 n_i m_i}}$$

Toroidicity Induced Alfvén Eigenmode (TAE)



$$\omega_{TAE} \sim \frac{v_A}{2R}\left(\frac{n}{m+1/2}\right) \propto \frac{B}{\sqrt{n_e}}$$

Reversed Shear Alfvén Eigenmode (RSAE)

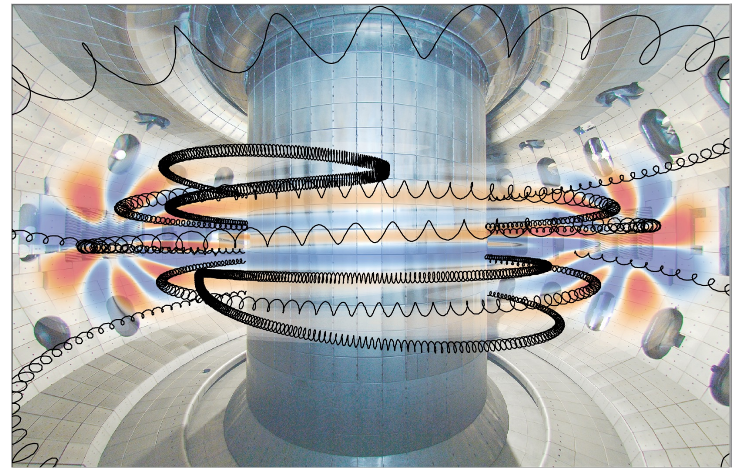


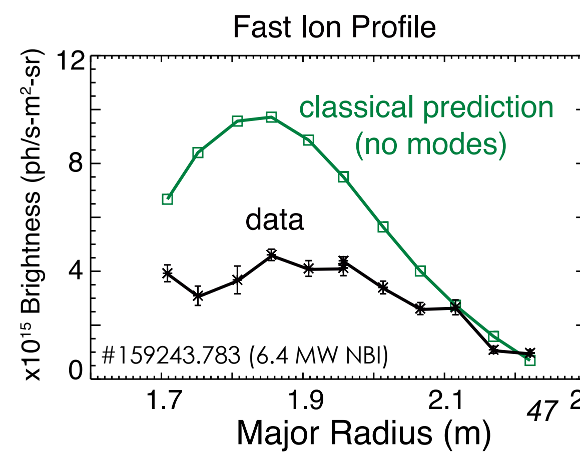
$$\omega_{RSAE} \sim \frac{v_A}{R}\left(\frac{m - nq_{min}}{q_{min}}\right) \propto \frac{1}{q_{min}} \frac{B}{\sqrt{n_e}}$$

C. Collins AAPS (2018)

Alfvén eigenmodes (AE) can cause fast-ion transport

- Fast ions created through injecting energetic neutral particles, ion cyclotron resonance heating, or fusion reactions
- AEs are MHD instabilities driven by wave particle interactions
- In DIII-D, high beam power can drive strong AE activity, causing fast-ion profile to flatten





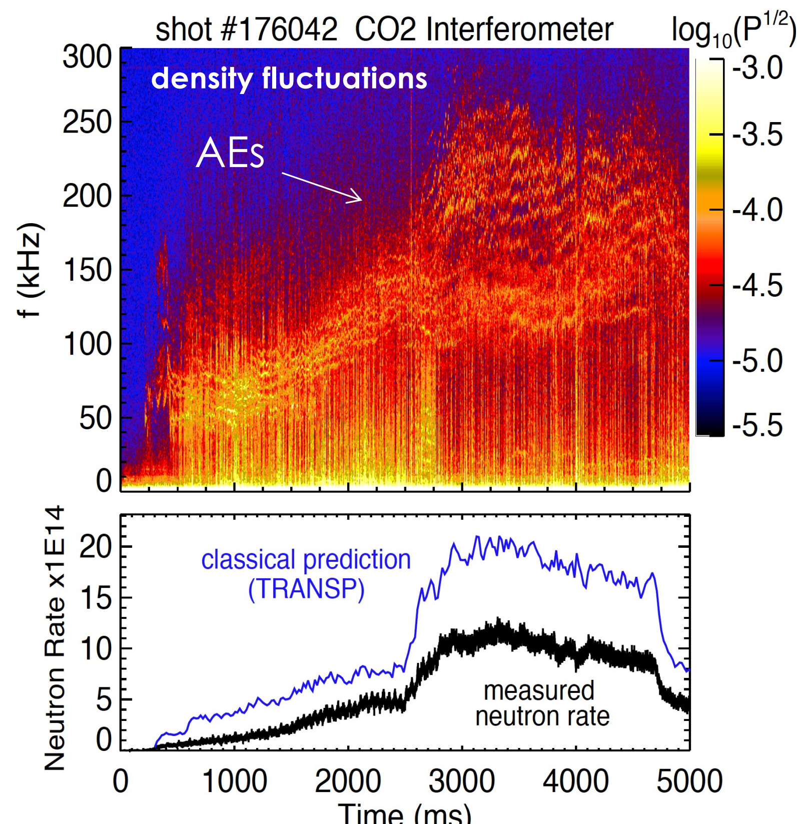
[Heidbrink et al., PRL 99, 245002 (2007)]

Fast-ion transport can reduce fusion performance and lead to losses that damage fusion reactor walls

- AEs cause transport that can:
  - Reduce absorbed beam heating power
  - Reduce current drive
  - Reduce achievable β_N (fusion power $\propto (\beta_N)^2$)
  - Cause fast ion losses that damage walls

- A ‘sea’ of AEs are predicted to be unstable in ITER

- Important questions:
  - When is transport significant?
  - What can we do to control AE transport



[Heidbrink, PPCF 56 (2014)]

[Holcomb, PoP 22 (2015)]

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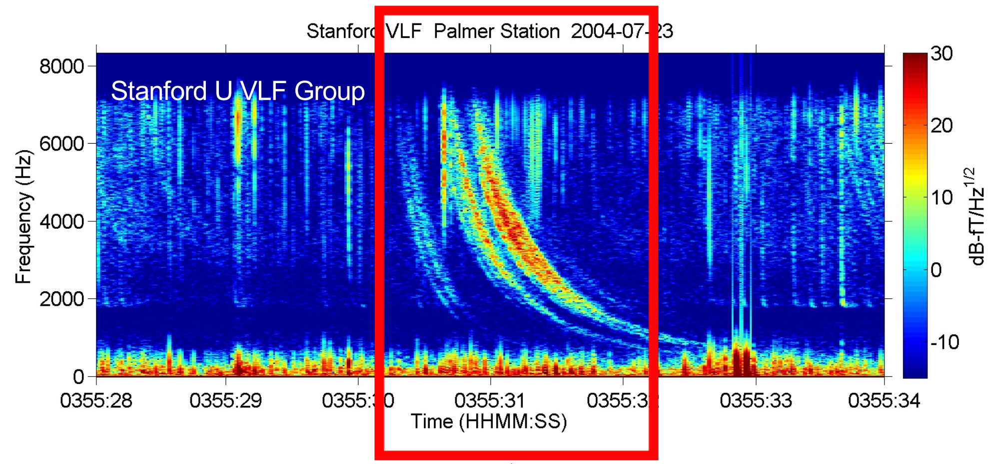
  - **Whistler waves**

- Propagation perpendicular to B_0
  - Ordinary and extraordinary waves

Whistler waves found in magnetosphere

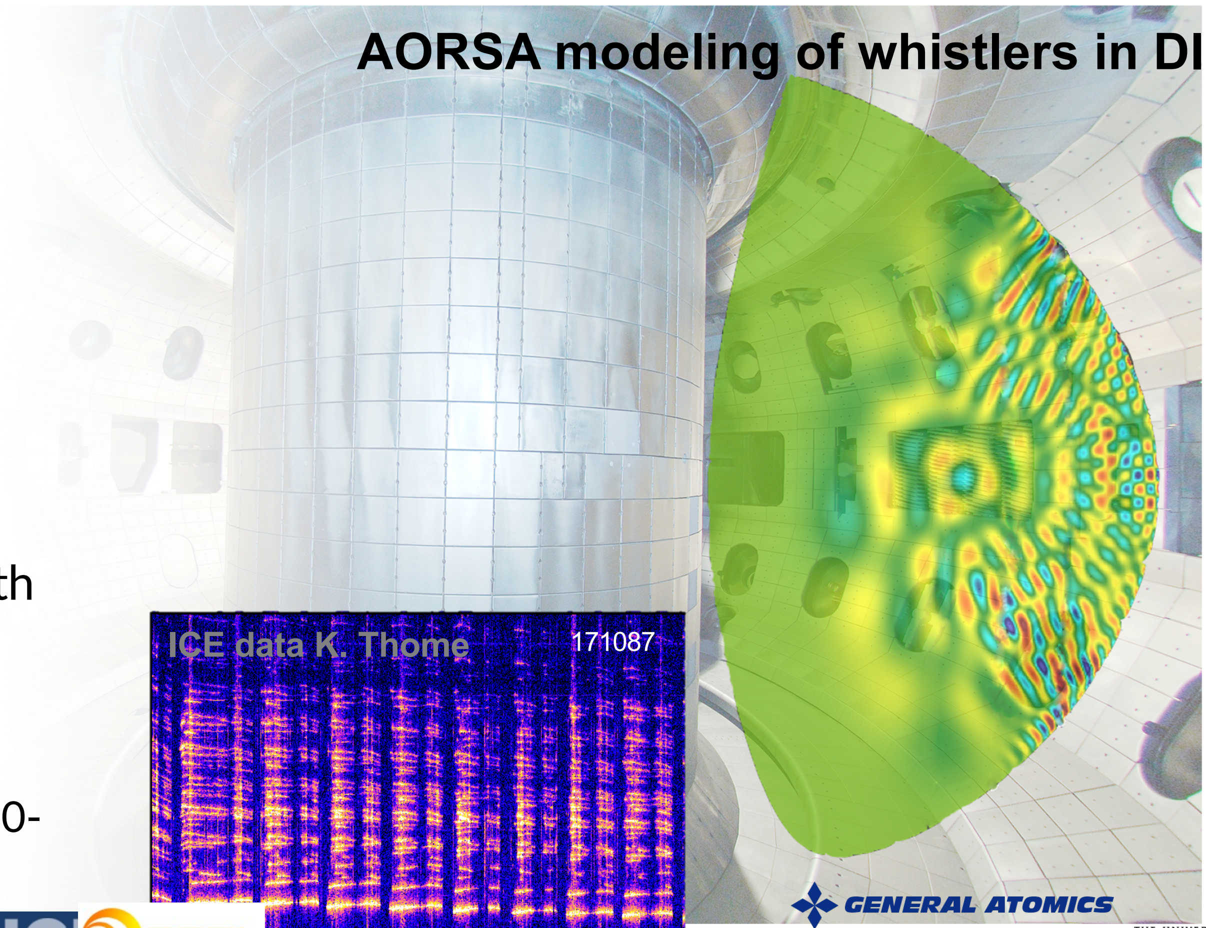
- Originally observed by radio/telephone operators in WWI/II
- Lightning strikes excite broad range of radio frequency waves in magnetosphere
- Some whistlers born at strike site, propagate along earth's dipole field
- Because of dispersion, higher frequency waves go faster than lower frequency
  - Phase and group velocity depend on frequency



Whistlers heard as a descending tone, higher frequencies propagate faster along magnetic field

Whistler waves observed in tokamaks

- Runaway electrons provide driving energy for whistler waves
  - Increasing B suppresses whistlers
  - Decreasing B enhances whistlers
- Observed more whistlers with increased intensity in measured hard x-rays
  - Dispersion relationship suggests electron energy ~10-15 MeV



K. Thome RSI (2018)

D. Spong PRL (2018)

Waves in cold plasma dispersion relation

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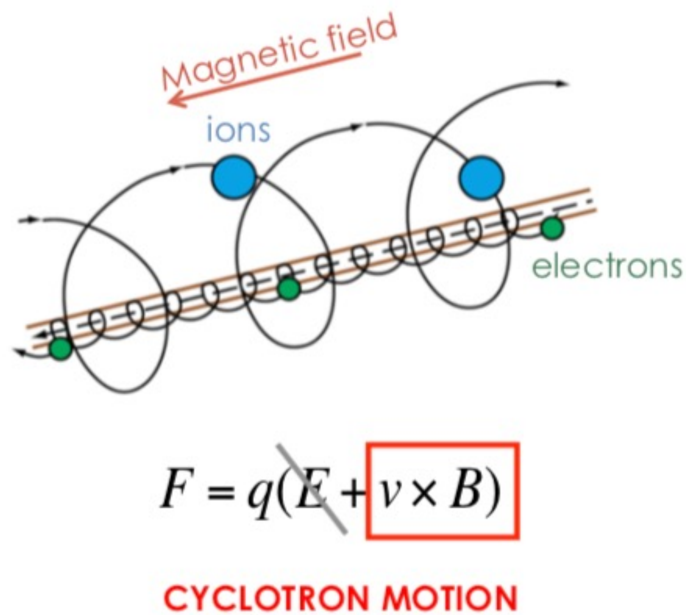
  - Alfven waves
  - Whistler waves

- Propagation perpendicular to B_0
  - **Ordinary and extraordinary waves**

Electron cyclotron range waves provide heating, drive current

- Electron cyclotron (EC) frequency is the natural frequency of rotation of electrons in magnetic fields $f_{ce} = \frac{eB}{2\pi m_e c}$
- Considering this frequency range, there are two solutions to the cold plasma dispersion relation
- Ordinary mode (O-mode)
  - E is parallel to B
  - Independent of B
  - Depends on density
- Extraordinary mode (X-mode)
  - E is perpendicular to B
  - Depends on B, n_e



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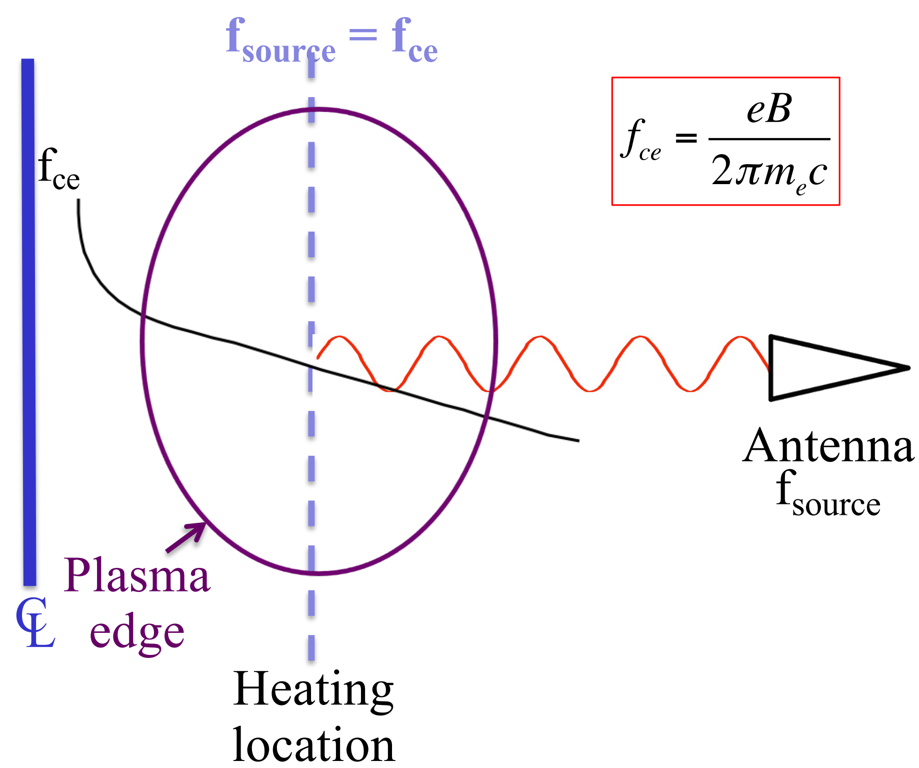
  - Independent of B
  - Depends on density
- Extraordinary mode (X-mode)
  - E is perpendicular to B
  - Depends on B, n_e

Both O-mode and X-mode have resonances at the electron cyclotron frequency, cutoffs depend on plasma density

EC waves provide localized heating/current drive

- Launched radiofrequency (RF) waves absorbed near cyclotron resonance
  - Tune to either electron or ion cyclotron motion
  - RF source frequency can be chosen to heat precise radius
  - For tokamaks, $B_t \propto 1/R$

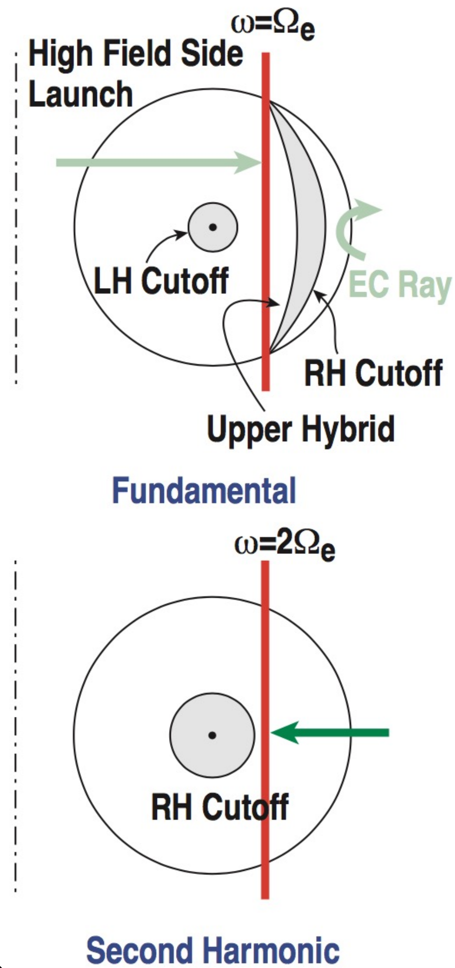


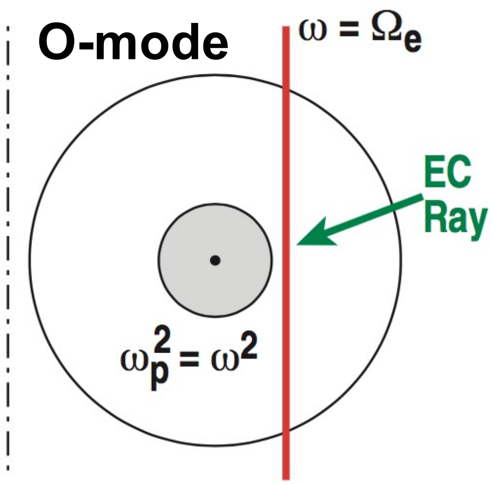
$$f_{ce} = \frac{eB}{2\pi m_e c}$$

EC waves provide localized heating/current drive

- Can provide:
  - Electron heating
  - Current profile control, sustainment
  - Control of magnetohydrodynamic (MHD) activity

X-mode

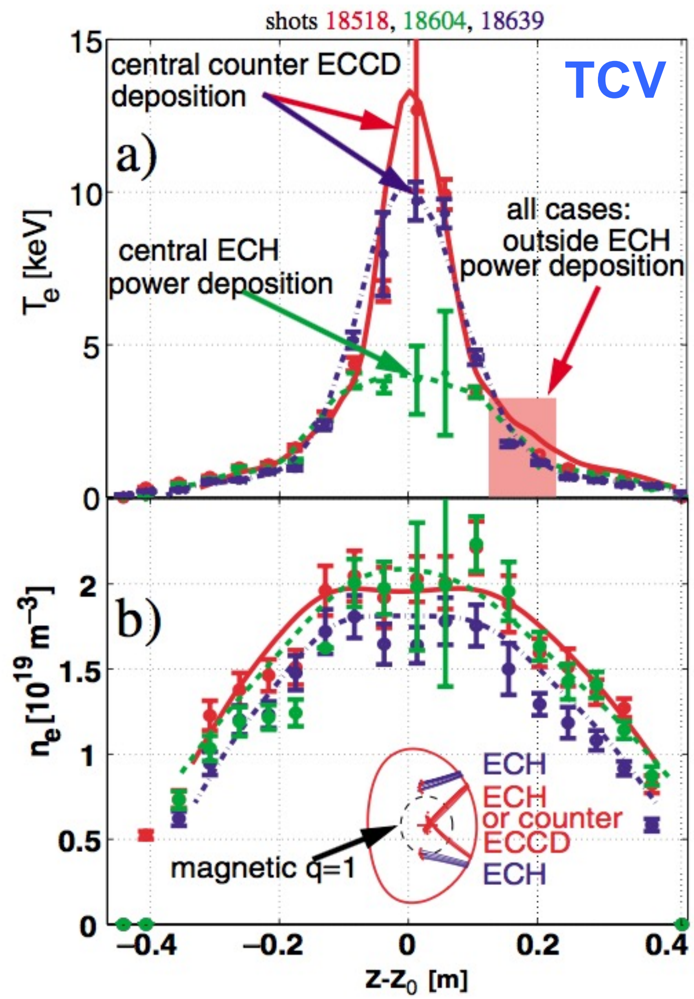




R. Prater PoP (2003)

EC waves provide localized heating/current drive

- Many examples of ECH/ECCD in tokamaks and other confinement devices
  - Large-scale, high-performance devices depend on waves for heating
- EC heating/ EC current drive can provide current profile tailoring in TCV
  - Improve central electron energy confinement
  - Stabilize MHD modes



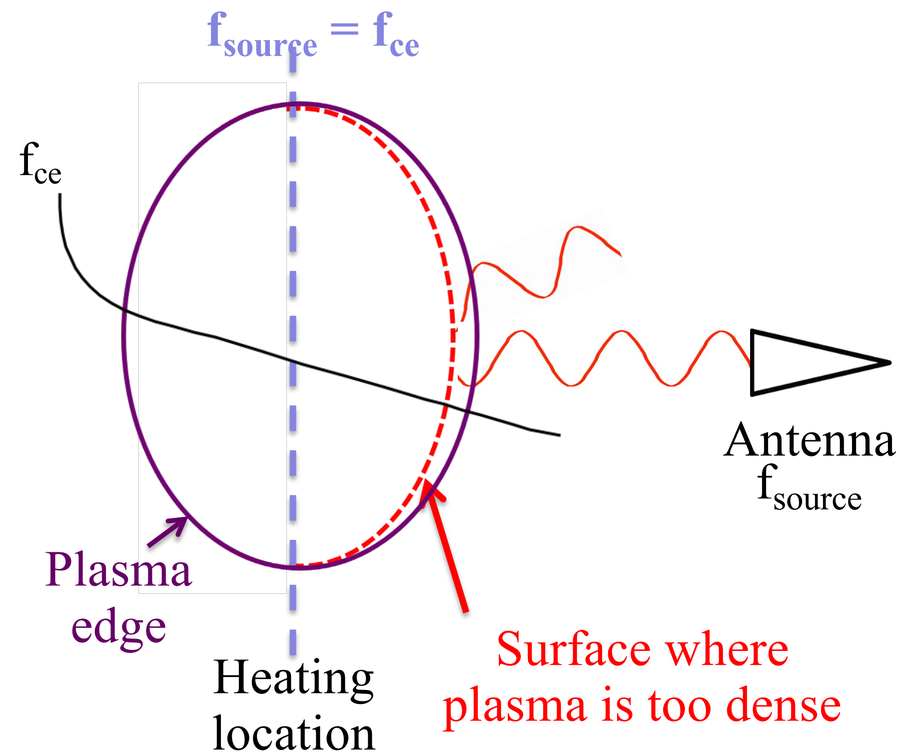
Z.A. Pietrzyk PRL 86, 8 (2001)

Electron cyclotron wave injection provides plasma heating, current drive – in certain conditions

- If plasma is too dense, O-mode and X-mode reflected near plasma edge
  - Happens in spherical tokamaks and stellarators

$$\omega_{source} > \omega_{pe}$$

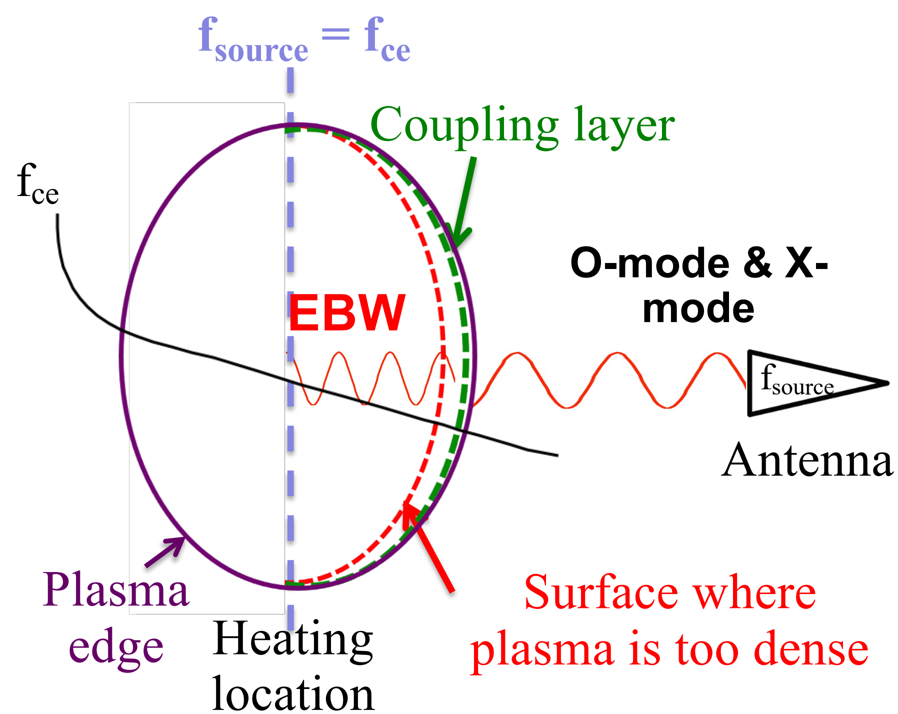
- Alternative heating method required



Electron Bernstein waves can travel in high density plasmas

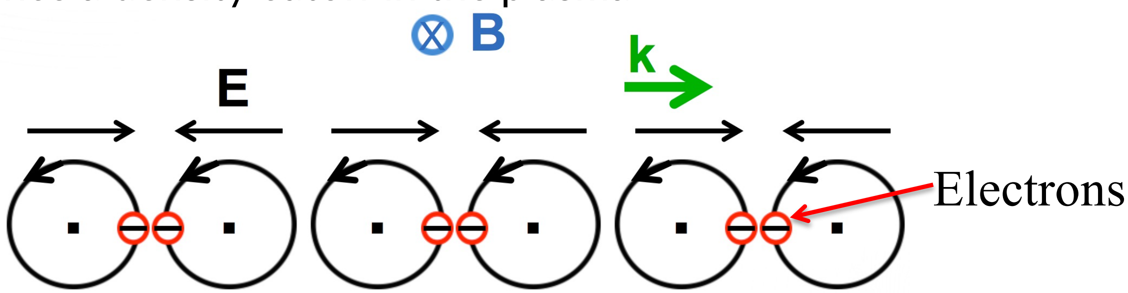
- Electron Bernstein Waves (EBW) can only travel inside the plasma
  - Wave moves due to coherent motion of charged particles

- Can only couple to EBW by launching O- or X-modes



Electron Bernstein waves can propagate in overdense plasmas

- Electron Bernstein waves (EBW) are hot plasma waves:
  - Longitudinal, electrostatic waves
  - Propagates perpendicular to B
  - Do not experience a density cutoff in the plasma



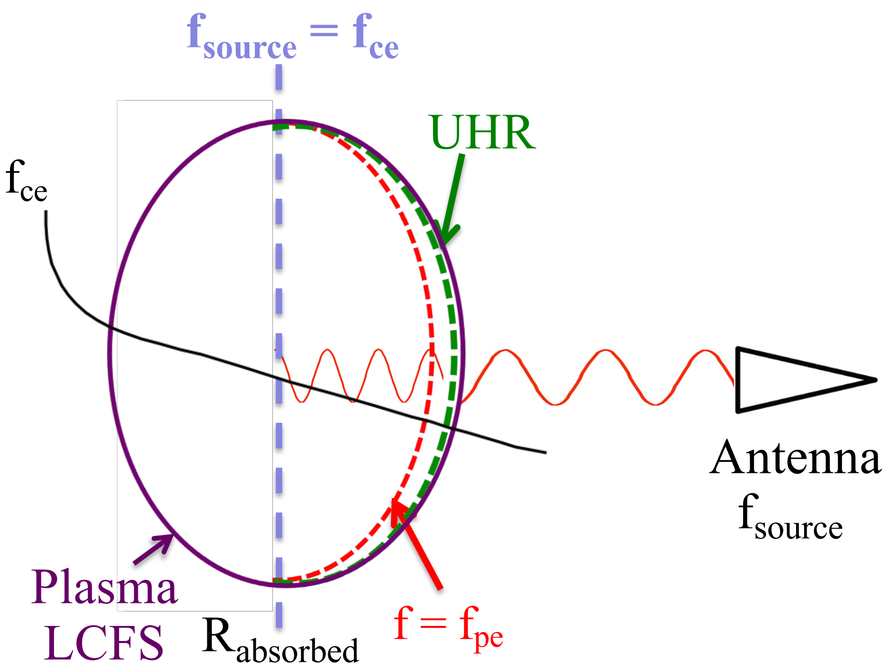
- Cannot propagate in vacuum -> must launch O- or X-mode to mode couple to EBW

$$1-2\sum_s \frac{4\pi n_s m_s c^2}{\lambda B_0^2}\left[\sum_s e^{-\lambda} I_n(\lambda)\frac{n^2}{\left(\omega/\Omega\right)^2 - n^2}\right] = 0 \quad \text{Where: } \lambda = \frac{k_\perp^2 \kappa T_\perp}{m\Omega^2}$$

- As wave frequency approaches EC harmonic, $\omega = n\Omega_C$, wave is strongly absorbed

EBW emission can be used to measure temperature

- Electron Bernstein wave emission at blackbody levels, proportional to local T_e



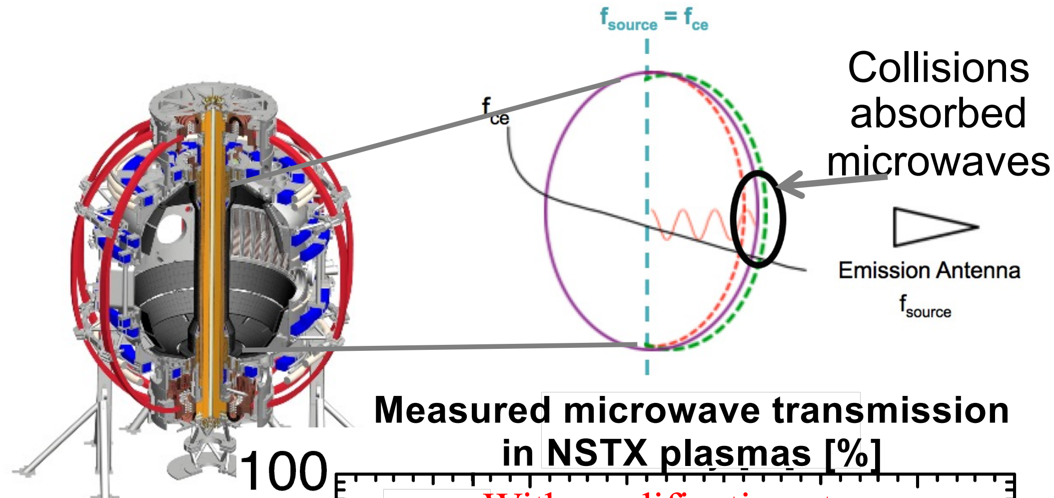
Larmor formula: $I_\omega = \dfrac{\omega^2 k_B T_{rad}}{8\pi^3 c^2}$

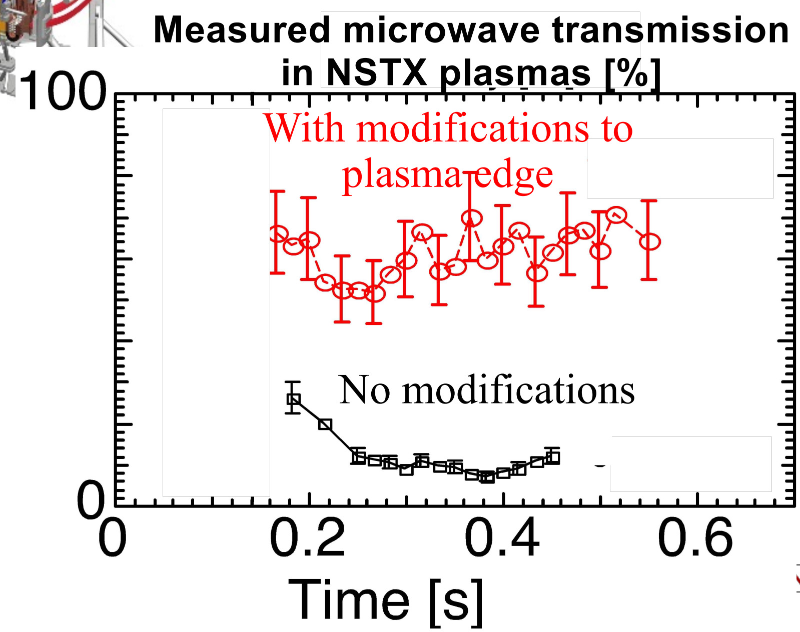
- Measured T_{rad} is proportional to local T_e
  - $f_{ce} \sim 1/R$ ➔ radial localization

Diem et al, PRL (2009)

Coupling microwave power to high density fusion plasmas can be difficult - but possible

- Plasma naturally emits microwaves from cyclotron resonance location
- Assumed physics of microwave emission from high density plasmas same as launching
  - Measurements on NSTX didn't agree with predictions
  - Plasma edge had too many collisions, absorbed microwaves
- ***Unexpected results present opportunities***





PEGASUS-III – lighting a match for fusion

- Future spherical tokamaks call for solenoid-free operation
  - Need to minimize solenoid due to shielding/cost
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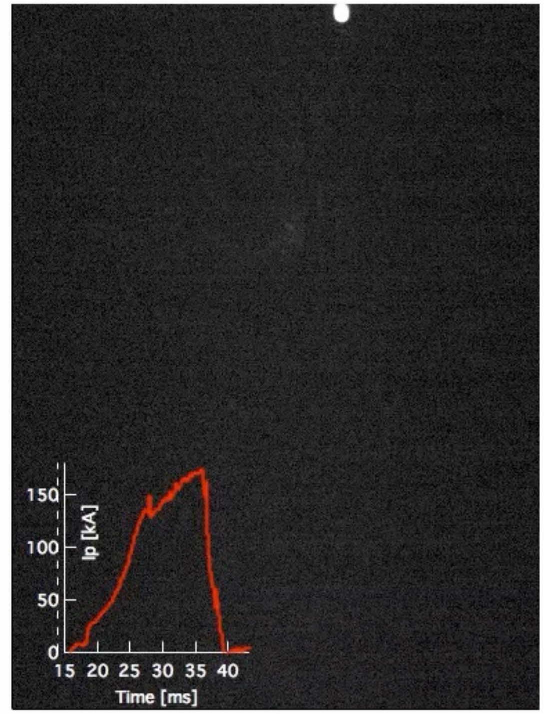

PEGASUS Website: Publications, Presentations
http://pegasus.ep.wisc.edu

PEGASUS-III – lighting a match for fusion

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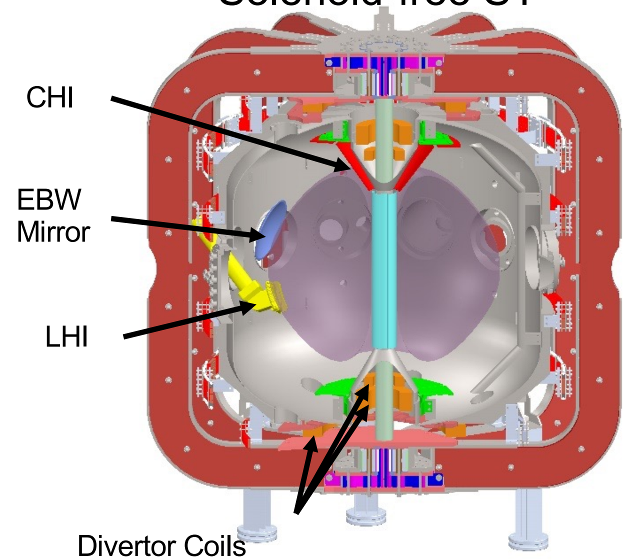
PEGASUS-III
Solenoid-free ST



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Plasmas support wide variety of wave phenomena

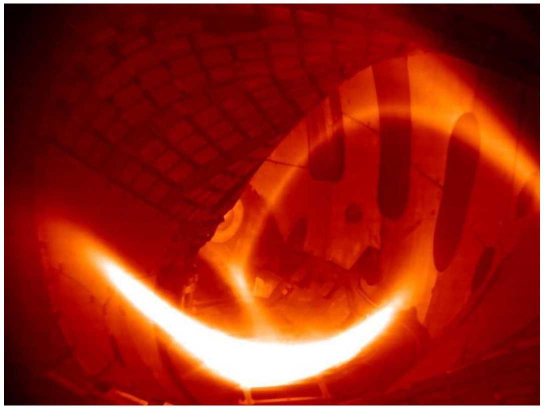
- Waves found naturally in plasmas
  - Described by dispersion relation

- Waves can deliver energy-momentum in plasma

- Waves can be used in plasma diagnostics

- Waves can drive turbulence…

Photo pf aurora: Senior Airman Joshua Strang



First W7-X plamsa, IPP, Greifswald

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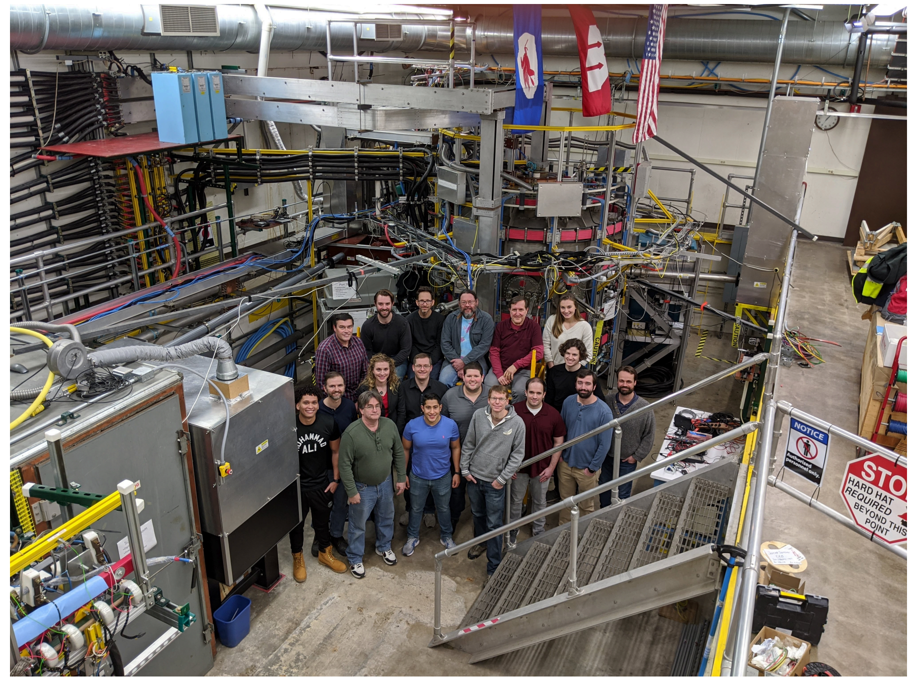

Contact me, Steffi Diem (sjdiem@wisc.edu), for research and educational opportunities on PEGASUS-III